Sign and Fuzzy Automata

The formal analogy between the definition of sign and the system of relations defining an abstract automaton (Mealy, 1955) was suggested by Max Bense (1971). Of course, it is possible to proceed in this direction and develop a generative semiotics (Nadin, 1976) studying, for instance, the problems of minimization, that is, problems of the repertory. It is not within the scope of this study to present such results (see also Tomescu, 1971)—even if we consider them very important—but to propose the extension of the analogy from abstract automata to fuzzy automata. In order to do so, we must proceed as follows:

1. Demonstrate that fuzzy functions are suitable to the sign in Peirce's semiotic;
2. Introduce the definition of the fuzzy automaton (and a fuzzy Turing Machine);
3. Determine whether the analogy is formal and to which extent.

It was already shown (Nadin, 1976) that, at least in principle and to a certain point, the sign relation might be considered an intersection in the terms of set theory, or rather, fuzzy set theory. Already a problem of principle arises: If the fuzzy functions, as introduced by Zadeh (1965) and which represent the "grade of membership" (Menges, Skala, 1974) to a set (e.g., the set called Repertory) are consistent with Peirce's semiotics. In order to prove this, let us first present the general theory of *fuzzification,* the result of which is the expression of inexactness (quality, continuity) in mathematical terms.

A fuzzy function (or fuzzy relation) denoted by (a)

$$f : X \quad Y$$

from X in Y, for instance, from the set of signs *S* to the set represented by the repertory, i.e., the sign as a sign or ground, in Peirce's terms, is the fuzzy subset of the product X x Y. Thus (b):

$$f : X \times Y \rightarrow [0,1], \text{ or } f \in F(X \times Y)$$

in which f(x, y) is the degree of membership of y at the image of x by f (or, "the intensity of the relation between x and y." The so-called "ensembles flous" work on the same basis; Gentilhomme, 1968).

The extent to which a sign is to be considered of a particular type (Legisign, Sinsign, Qualisign or Index, Icon, Symbol, etc.) is determined not through a two-valued (or even three-valued) characteristic function (Lukasiewiczs' type of logic), but through a fuzzy relation. Peirce was aware of this. But before recalling his conception, an observation: Not only the repertory relations, which could be represented through set theory, but also the object relations, requiring categorical algebra (cf. Peirce's student, MacLane) or the interpretant relations, requiring Neumann"s *number theory,* can be approached from the same fuzzy perspective (Goguen, 1968).

The treatment of the sign in the terms of fuzzy set theory corresponds to its nature as defined by Peirce. The sign is a reality in itself (sign as sign), but "No sign can function as such. . . ." ". . . it is absolutely essential to a sign that it should affect another sign." (CP 8.225). (This was the subject of my paper *The Integrating Function of the Sign in Peirce's Semiotics,* submitted to the C. S. Peirce Bicentennial International Congress, Amsterdam, June 16-20, 1976 and read by a proxy since the communist government in Romania would not give me a passport.) In his attempt to divide semiotic—pure grammar, logic proper, pure rhetoric—Peirce already used a fuzzy (before the term was coined) terminology: "What is quasi-necessarily true of the representamen," (CP 2.229). Knowledge itself ("every species of actual cognition is of the nature of a sign," CP 7.355) is doubly articulated as *processuality:* "The whole process of representation *never* reaches a completion" (CP 1.873); and *continuity,* "governs the whole domain of experience in *every* (italics mine) element of it" (CP 7.566). All knowledge is dominated by a fuzzy concept, *synechism* ("that tendency of philosophical thought which insists upon the idea of continuity as of prime importance in philosophy," CP 6.169).

For a while, Peirce was absorbed in the problem of the representation of the continuum through discrete signs. He introduced the concept of potential collection (CP 6.187), "indeterminate, yet capable of determination" (CP 6.185), as well as the concept of "vague" (CP 6.186). "It is vague, but yet with such a vagueness as permits of its accurate determination in regard to any particular object proposed for examination." I consider the *primipostnumeral multitude* (CP 4.211*)* to be an anticipation of fuzzy sets. Finally, in order to undertake the last step towards the consideration of the analogy between one of the sign's definitions and the definition of a fuzzy automaton, it should be noticed that the sign proper contains, in Peirce's view, an internal self-adjusting system, introducing itself as such a unit.

"The object of representation can be nothing but a representation of which the first interpretation is the interpretant . . . So there is an *infinite* regression here. Finally, the interpretant is nothing but another representation to which the torch of truth is handled along; and as a representation, is has its interpretant again. So, another infinite series," (CP 1.338).

The definition of the sign through three non-empty sets M, 0, I, and the two operations on those sets, *o* as the *transfer* function and *i* as the *accomplishing* one (c)

$$S = S(M, 0, I, o, i)$$

has an obvious analogy to the definition of an abstract automaton given through a quintuple as follows (d):

$$A = A(X, Y, Q, \delta, \lambda)$$

in which

X = the finite set of inputs
Y = the finite set of outputs
Q = the finite set of states
δ: 0 x X $\rightarrow$ Q = the next-state function (transition function)
λ: Q x X $\rightarrow$ Y = the next-output function (output function)

We shall recall the observations occasioned by the analogy suggested by Bense (1971): A sign as a sign (ground) is a system of states, i.e., of possibilities, determined by the object for which the sign will stand (the object as input). A sign accomplishes its sense (Bense says *Bedeutung;* while Peirce distinguished between sense, meaning, significance) only as fulfillment, as interpretation "cognition produced in the mind" (CP 1.370) (the output). I shall add (Nadin, 1972): A generative semiotics is in fact generation of sense, the later defined as the quality of a content (with the nature of the significance), semiotic processes being *significative* in the first place, and thus also *communicative.*

Before proceeding to the analogy, which I consider not exclusively formal, between the sign's definition and that of a fuzzy automaton (application of fuzzy sets to system analysis), it is useful to suggest that the problem of minimizing abstract automata is in fact related to the sign's realization, i.e., semiosis.

Given an automaton M, minimizing the number of its states (minimizing the repertory that will generate a given sense, or a couple of them) means finding another automaton having the property (e)

$$M' \supseteq M$$

for which the number of states is a minimum (contains a minimum number of elements. This, in fact, is the most restricted repertory of signs leading to a sense to be generated. The problem can be resolved through varied algorithms, if necessary using a computer for the calculation. We will not approach this subject now preferring to extend the analogy between the sign's definition and that of a fuzzy automaton (FA).

A FA (Moore type, cf. Negoita, Ralescu, 1975) is also represented by a quintuple in which the sets X (inputs), Y (outputs) and Q (states) are finite. The functions δ and λ, i.e., the *dynamics* and the *output map,* are fuzzy relations (e):

$$\delta : X \times Q \times Q \rightarrow [0, 1]; \quad \delta : X \times Q \quad X$$
$$\lambda : Q \times Y \rightarrow [0, 1]; \qquad \lambda : Q \quad Y$$

If the initial state is represented by $q_0 \in Q$ (a sign determined from a repertory of signs), the fuzziness of the system is due to its functions. This is the case of the generation of an equivocal sense. We may also consider here the initial state as a fuzzy subset of Q (the fuzzy subset of a repertory

$$\xrightarrow{\text{fuzzy}}).$$

In this case, the initial state is a fuzzy vector (f): 1.1, 2, 3

$$P_0 = (i_1, i_2, \ldots i_n) \quad (K)$$

in which $i_j \in [0, 1]$ is the degree of membership of the state $x_j \in X$ at the fuzzy initial state.

One example is the signs of the repertory in visual poetry or in action writing. As a result, we have the ambiguity (plurivocity) of the sense embodied in such a process and which is its specific goal. Following the same path, the dynamics itself can be viewed not only as a fuzzy relation, but as a family of fuzzy matrices over [0, 1].

For

$q_i, q_j \in X$

we shall denote

$\delta q_i, q_j(x) = \delta(q_i, x, q_j)$

and thus

$x \in X \longrightarrow T_x \in FM(x), \quad T_x = (\delta_{q_i,q_j}(x))$

Obs. $1 < i,j \leq n$

A fuzzy matrix (FM) — or a matrix over $\mathcal{S}$ (an ordered semiring) — is a function (26)

$$\mathcal{A} : \{1,2 \ldots m\} \times \{1, 2 \ldots n\} \rightarrow \mathcal{S}$$

or,

$M = (m_{i,j})$ is called an FM if

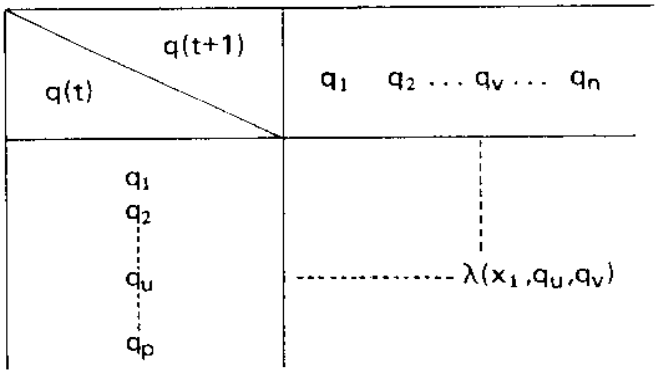
$o \leq m_{i,j} \leq 1$

Usually, the functions δ and λ are given through FM. So, if at the moment t the input is x_l, then

$$T^t_{x_l} = (\delta^l_{i,j}(t))_{i,j}$$

and $\delta^l_{i,j}(t) = \delta(x_l, q_i, q_j) = \delta(x(t))$

$\lambda_{i,j}(t) \equiv \lambda(q(t))$



Every event that can be represented in Mealy's type of finite automata is regular events (cf. Marcus, 1964). The same holds for events represented by fuzzy automata. Semiotic processes are regular. A sign's derivation from another sign, which is expressed by the functioning of the fuzzy automata, is not only a generative process, but also explains the structure of the process and has an analytical opening. A finite automation is the type most peculiar to a Turing machine. The fuzzy Turing machine (FTM)—

also defined through a quintuple analogous to that of a sign (g)

$$T = \{A, B, X, \delta, i\}$$

in which

A = the printing alphabet
B = the auxiliary alphabet (special symbols)
X = the set of internal states
δ: X x U x V x X $\rightarrow$ [0, 1] the transition function
i: X $\rightarrow$ [O, 1] the fuzzy initial state

and

$U = A \cup B$; $V = U \cup \{+1, -1, 0\}$,

in which +1, -1, 0 respectively represent the possible moves to the right, left or the procedure's end—

has the condition of an algorithm, not that of a sign. The behavior of an FTM is analogous to that of an FM and suggests the analogy between sign processes and learning processes.

We can benefit from the definition when we recall that Turing machines TM are distributed between TM free of any restriction and TM as finite automata. The weakest condition that can be imposed upon a grammar is to include it in the class of TM free of any restriction. The most restrictive—realized by the sign, as we saw—is to be a Markov source, i.e., a finite automaton, in this case a fuzzy finite automaton (FFA). Peirce considered that "the highest grade of reality is only reached by signs" (CP 8327). The inferential nature expressed by the characteristic quintuple defining both the sign's definition and that of an FFA belongs also to Peirce's philosophy, and should be emphasized as such: "every state of consciousness [is] an inference, so that life is but a sequence of inferences or a train of thought" (CP 7.583).

The sign has the nature of a universal and can be evaluated only through another sign, an idea extended to man himself: ". . . as thought [the sign] is a species of symbol, the general answer to the question 'what is man?' is that he is a symbol. To find a more specific answer we should compare man with some other symbol," (CP 7.583). And this comparison is a definition of the type of semiosis belonging to what we shall call *axiological semiotics* (Nadin, MS).

The processes of semiosis should be considered, in their generality, as fuzzy. A final term to be introduced and determined is *sense* (meaning, significance). Considering *U* a universe and *N* a set of signs applied as labels for the fuzzy subsets of *U,* we can define the sense $\Sigma(x)$ of a term $x \in$ N, as the fuzzy subset of *U* defined by the function (law of composition)

$$\chi_{\Sigma_{(x)}} = \chi_x$$

The degree to which a sign fulfills a Rhematic (3.1) sense, or stands for a Dicisign (3.2) or for an Argument (3.3) is expressed through the above-mentioned function. Peirce (CP 8:314, 8:315) defined three fields of the interpretant: *immediate* ("the Quality of the Impression that a sign is fit to produce"); *dynamic* ("whatever interpretation any mind actually makes of a sign"); and *final* (it "does not consist in the way in which any mind does act but in the way in which every mind would act"). The idea of self-control ("conduct" as "action under an intention of self-control") is also expressed. The triadic and trichotomic division of signs is confirmed by an FA's definition (of course, considering

$$\underset{c=3}{=}$$

as defined for Peirce's semiotic, Nadin 1976). The sense embodied can be emotional (affective), energetic, or logical.

Let us consider an example: the nucleus K comprising also the set of numbers representing the wave's length (in Å) of the visible spectrum

$$K = (0, 1, \underbrace{\ldots\ldots 4000 \ldots\ldots\ldots}_{\text{visible spectrum}} 7000)$$

and the universe U = K

The set T = {red, orange, yellow, green, blue, indigo, violet}of the colors of the visible spectrum (light) is fuzzy. The sense (physical) of $\underline{x}$ is $\Sigma(\text{green}) \subseteq U$, a fuzzy set having χ_{green} as its composition law. This law can be given as:

$$\text{green} \begin{cases} 1+[(n-5.10^3)^{-2}]^{-1} & \text{for } n > 5461 \\ 0 & \text{for } 5000 > n > 5700 \end{cases}$$

Green as a fundamental being defined by green = 5461 Å, (or by a more sophisticated spectrometric definition), we say that the sense above mentioned is the *physical* one. Even in this case it is obvious that the sense is defined in connection to colors and light. It should be recalled that the physical sense of any color is given by the property of radiant energy that permits the eye to differentiate between two neighboring sections, identical and equally illuminated, of a homogeneous surface, so that if the two sections do not appear as distinct, they have the same color. The physical sense itself (energetic) can be enriched by considering nuance (given through λ), purity (depending on the degree of mixing with the color white), and brightness (depending on the radiating surface and on the spectral sensitivity of the eye). Green is a sign *per se* for a physicist (1.3); in painting, it is a *Qualisign* (1.1). If, in painting, it is also a special shade of green—a painter's "secret" (pigment, brightness, brilliance, etc.)—it becomes "fuzzy," a *Sinsign* (1.2). The radiating surface (wood, canvas, cartoon, metal, etc.) contributes to its quality (1.1). As a standard green (determined strictly on a spectrometric basis, or given in a catalog) it is even a *Legisign.* The same goes for the object and interpretant relations (green as an "object" belonging to the "thematics of sign," green as "cognition produced in the mind"). Of course, the nuances—light green, Lincoln green, dark green, emerald or olive green are distributions and participate in the *endowment* of sense, meaning and significance, i.e., in the processes of signification (comprising communicative processes), in the same way. In an iconic representation, the color can be used pure or mixed. In an action painting, it fulfills mainly an indexical function, but a symbolic or even iconic function (of the action) can be considered. The sense accomplished by a color ("quality of the impression") in a painting depends upon its relations with neighboring colors, juxtaposition, the way of setting it on canvas (or whatever the painting's support may be). It follows that sense (fuzzy) is realized not textually, but in the context following both the aforementioned composition laws and the structural determination given by the main triadic sign relation (and also trichotomic division). Any other type of composition law can be imagined within the scope of the use of color (or of a form, sound, etc.). A green circle embodies a sense different from a green acute triangle (according to Kandinsky).

We can, of course, refine our analysis, pointing out the ambiguity generated through a sign process. If we consider the nucleus

$$X = \{\text{visible objects}\}$$

and the set of colors

$$T = \{\text{colors, white}\} = \{r, o, g, \ldots v, w\}$$

we understand that the sense of red, orange, green . . . (the Fauvists instilled the sense of pure colors) i.e.

Σ(red), Σ(orange) etc. are subsets of K and therefore concepts of mediate ("secondness", in this case) level. Even the sense of "more red than", ($\Sigma\uparrow$ red) is of the same level

$$(\Sigma\uparrow (\text{red}) \supseteq K^2).$$

The following concept, corresponding to a higher degree of semiosis (from the thematics of reality to that of the sign) is "colored" (supersign) and it corresponds to a sequence of signs of higher semioticity:

Σ(red), Σ(orange), . . .

It is of the level of a "thirdness". Higher levels follow, but always in the basic triadic structure as given by Peirce (c=3). Retrosemioses are to be defined in the same way. It is possible to give similar examples concerning the universe of sounds or shapes, of course considering the specific way in which sense (and the sense of sense, etc.) is established and then participates in axiological processes (as their object, or their mean).

The analogy between the sign's definition and that of a fuzzy finite automaton, makes sense (surpasses the level of formal analogy) only if we consider the characteristic sets of a triadic sign relation as having a fuzzy *implicit* nature. Moreover, that is the way in which signs participate in the formation of languages (verbal or not). We shall not elaborate here upon the problems concerning the languages generated by fuzzy automata and fuzzy grammars (in fact forms of syntactics that as such belong to semiotics). But it should be emphasized that such a topic is of main importance for a consistent theory of signs. However, if the output of a system such as that represented by the sign is a partially ordered set, this can be viewed as a fuzzy language embodying the fuzzy sense (meaning, significance, as sense of sense, sense of sense of sense) to be instilled.

The same fuzzy automaton (and the language accepted by it, its fuzzy grammar) also implies, as stated above, the principle of the sign's degeneration mentioned by Peirce. The passage from signs of higher semioticity (defined by Bense as *bewßtseinsunmittelbarer*, or consciously direct) to lower *(weltunmittelbarer*, worldly direct*)* semioticity is also expressed, and his model of the sign's ten main trichotomies is directly confirmed and can be calculated. Not all 59049 "classes of signs" (3^{10}) "prove to be independent of one another." The "triple connection" of signs: "sign, thing signified, cognition produced in the mind" remains however fundamental and explains also the ten trichotomies (cf. Walther, 1976). More about the subject can be discovered if we consider also the problems of minimizing fuzzy automata. For this we must determine all the pairs of *compatible states* (comparable to what Bense discovered as "Dualisierung" or dualizing) of the automata and to build a so-called cover C_i for them. In principle, it is a solvable problem, but it implies refined mathematical tools.

Consequently, compatibility of states (repertory relations) must be defined. Peirce's trichotomic model is, in fact, the first conceptual attempt at minimizing of the sign's fuzzy automaton. In the practice of sign use through which man also becomes a "semiotic animal" (Mongré, 1897), minimization is implicit to sign processes. Its complement (maximization, i.e., instillment of ambiguity as such, is equally assumed as a goal in art and philosophy, for example.

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