

## The logic of vagueness and the category of synechism

In his article “Issues of Pragmaticism” published in 1905, in *The Monist* (vol. 15, pp. 481-99), Charles S. Peirce complains that “Logicians have been at fault in giving Vagueness the go-by, so far as not even to analyze it.” That same year, occupying himself with the consequences of “Critical commonsensism,” he affirmed, “I have worked out the logic of vagueness with something like completeness,” a statement that causes the majority of the commentators on his work, including the editors of the Collected Papers [1] to ask where this logic is to be found. The fever for finding Peirce’s manuscripts is fed by the hope of some researchers of discovering the logic of vagueness, a hope that has grown since Carolyn Eisele’s publication of his mathematical works. Others-and I count myself among them-believe that in reality this is matter of something already known. That is, they interpret the affirmation ending the paragraph of reproach addressed to logicians, “The present writer has done his best to work out the Stechiology (or Stoicheiology), Critic, and Methodeutik of the subject,” (i.e., Vagueness) as a tripartite semiotic of the vague, still limited, according to Peirce’s older works (1896, “Preface” to *The Simplest Mathematics*), to symbols, that is, to the signs of natural language examined from the perspective logic.

If Peirce could read how much is written today on the type of problems he had in mind when he used the term “vagueness,” he would surely clarify himself. But at the same time, he would surely clarify himself. But at the same time, he would observe that in the place of a logic of the vague, more types of semantics of linguistic imprecision have developed that either ignore him as a possible predecessor-and not the least among the reasons for this would be, in addition to the unfortunate manner in which he was published, his vague mode of expression-or cite him inappropriately.

In connection to all these problems, and especially in direct relation to this very theme, I intend the following:

1. to consider vagueness in connection to the general vision of Peirce’s philosophy, examining the logic of vagueness
  - a) in the context of the epoch and
  - b) from the perspective of the present;
2. to explain vagueness in the context of the current semiotic tendency, that is, it epistemological and not gnoseological aspects, a problem of pragmatics and not semantics;
3. to attempt to establish the reciprocal relationship between the logic of vagueness and the category of the synechism (the principle of the continuum) and the way in which they lead to fuzzy types of logic.

### Historical Remarks

Peirce’s standpoint, according to which vagueness is a question of representation and not a peculiarity of the object of the representation is clearly stated: “. . . reality is something entirely definite,” (MS 385, p.A). This position is obvious in relation to the representation’s nature as process (infinite). Hence, it derives from the gnoseological angle. Later, Russell, from the same angle, stated the same idea: “Apart from representation, whether cognitive or mechanical, there can be no such thing as vagueness or precision” [2] giving as an outright condition for a perfectly

logical language that it should in no way be vague. Insistence on the gnoseological moment in definition of the real as object of representation partially explains the subsequent tendency to explaining vagueness in the terms of the theory of knowledge. Frege [3], about whom it has been lightly affirmed that he did not understand the need for a special logic of vagueness, has the double merit of having grasped what he called *die Weichheit und Veranderlichkeit der Sprache* (the softness and changeability of language, [4] that is, the determined inner nature of vagueness, which he wanted to limit through the aid of a *Begriffsschrift* (ideography) and the fact that these characteristics are a condition for the development of language.

The historical context can be enlarged, especially in view of the need to see to what extent a logic of vagueness is really possible and what accumulations have been made in the meantime accomplishing this. Analytical philosophy, for example, translates the question of vagueness into the space of extension and intension. Carnap [5] even gives a kind of negative definition of the zone of vagueness: "If an object  $y$  has neither the intension  $F_1$  nor  $F_2$  of the predicate  $Q$ , then a speaker  $X$  cannot even attribute, but neither can he not attribute, the predicate  $Q$  to  $y$ ." Wittgenstein [6] implied the idea of the gradual nature of similarities, hence, the continuity factor, which after Peirce had been lost sight of for a long time. Later developments were more and more specialized and technically refined. To name a few: Montague's meaning function [7], Quine's idea of vague as a consequence of the mode of learning expressions and meaning [8], Lewis's pragmatically relevant factors [9], Lakoff's distinction between vague boundaries and fuzzy hedges [10], etc. I emphasize, however, that they do not actually attain a logic of vagueness but only a logic of the production and determination of meaning. Vague is therefore considered as a semantic notion, fuzzy semantics attempting to give quantitative rules for specifying vagueness in the particular universe of linguistic discourse. The concentration on natural language as a particular sign system and the stereotyped repetition of the fact that its semantics is inexact still have not led very far, one of the very reasons why, in words different from Peirce's [11], "need for an exact theory of inexactitude" is more and more frequently called for and attempts are being made in this direction.

Taking the whole evolution into consideration, from the mere intuition of vagueness to the diverse formalisms, especially semantic, we cannot help observing that progress in the knowledge of the mechanism of producing meaning in natural language has been relatively minor and that limiting the problematic to double articulated language is still a step backward in relation to Peirce's concept of vagueness. The latter is a general theory concerning the relation between vague and determined as they appear in thought and communication processes and also in processes of signification viewed from the most general perspective regarding the sign, hence, without being limited to the linguistic sign. According to Peirce, thought is semiotic. It is dialogical and realized through signs. The sign itself cannot be absolutely precise. Its vague nature (indefiniteness) stems from the relation with the object of the sign which stands for that object or with its interpretant for which it brings about meaning (sense, meaning, signification). The first relationship (between sign and object) is a source of indefiniteness in Breadth; the second (between sign and interpretant) is the source of indefiniteness in Depth (cf. 4.543, 5.448). Peirce eventually dwells on one of these two types of indefiniteness: "Indefiniteness in depth may be termed vagueness, (MS 283, 141, 138-39, rejected pages). Peirce's early or more recent commentators (James Feibleman [12], W.B. Gallie [13], Ch. K. McKeon [14], J.L. Cohen [15], among others) did not keep this specification, so definitive of Peirce's concept, in mind. Of

course, it would be advisable to try to find out where the notions Breadth and Depth come from in order to more clearly understand even the abovementioned definition of vagueness. Peirce took over these terms from Hamilton [16] and initially applied them to the study of terms [17], for extension and comprehension respectively, proposing new meanings and even defining types such as informed breadth and depth of a term, essential and substantial breadth or depth of terms, etc. We can ask which of the types mentioned above participates in defining vagueness and thus arrive at the semiotic concept, initially presented as a theory of the symbol (“to include both concept and word,” 2.418) even though it is already the outline of the triadic structural model of the sign [18].

Vagueness concerns the informed depth, hence, “in a supposed state of information” (2.408). Moreover, according to Peirce, depth may be a certain or doubtful, actual or potential, which is also reflected by the types of vagueness (certain, doubtful, actual, or potential vague). The typology of vagueness was not, however, developed by Peirce and not even by those who later occupied themselves with it. W.J. Jevons [19], for whom breadth is extension and depth, intension, bewails, somehow in the same manner Peirce did when he wrote *Ethics of Terminology* [20], “the peculiar misfortune of the science of logic to have a superfluity of names or synonyms for the same idea. . . .” Jevon’s [21] use of the terms *breadth* and *depth* (terms not far in meaning from the same used by Peirce) is peculiar to the *pre*-semiotic period of logic (and of science in general). A sign is objectively general, in so far as, leaving to the interpreter the right of completing the determination for himself. A sign is objectively vague, in so far, as leaving its interpretation more or less indeterminate, it reserves for some other possible sign or experience the function of completing the determination,” (4.505). In his conception, the universal phenomenon of vagueness affects the logic of the non-contradiction, which Russell views differently, that is, that the law of the excluded middle is affected (a negative image, from possible true and false together to no true and no false). Peirce did not discover vagueness but only defined it as an implicit part of any sign process, linguistic or otherwise. Therefore, the consciousness of vagueness is part of semiotic consciousness, reflected by the latter in all of the forms of man’s semiotic practice.

### **Methodological Distinctions**

In order to understand the logic of vagueness which Peirce affirmed he had elaborated—an affirmation I believe to be true and justified—let us see what are the Stechiology (or Stoicheiology), Critic and Methodeutik of vagueness. They are the general theory of the nature and meaning of signs (viewed as representamina, hence icon, index, and symbol), the classification of arguments and the determination of their validity, and lastly, the study of the methods of investigation, exposition, and application of the truth (cf. 1.192, 2.93, 2.229, 3.430, and 4.9). From the view set forth in 1867—a reference date he himself gives—and until the one sustained in 1905, an evolution took place after which Peirce set up a *triadic-trichotomic semiotic* as a new type of logic of a universal nature. It necessarily derives from his general philosophical system, a system established on the basis of the phaneroscopic categories (Possibility, Reality, Necessity) and that it implies, as its very ordering principle, the law of synechism, that is, the doctrine of the continuum. The latter governs knowledge and implies generality. If all that exists is continuous (“Synechism is the doctrine that all that exists is Continuous,” 1.172), and if generality and continuity are the same thing (4.172), then we can

also understand why vagueness constitutes a universal principle and is not the result of a “defect in thinking or knowledge” (4.344), hence not a gnoseological accident. Vagueness can neither be eliminated (“vagueness . . . which is no more to be done away with in the world of logic than friction in mechanics,” 4.512) nor reduced to ambiguity, a danger that subsequent research has in fact not avoided.

As a semiotic animal (*zoon semiotikon*), man himself is identified as a sign and participates in the endless process of representing and interpreting reality. The potential infinity of the process of investigation and interpretation causes that only a relatively complete meaning (sense, meaning, signification) be determined at each moment. The process nature of knowledge concerns its epistemological condition. Vagueness hence represents a sort of relationship between absolute, final determination, which in fact is not attained (the condition of an ideal, therefore) and actual determination of meaning (again as sense, meaning, signification in concrete semioses. It can already be seen from the model of the processuality of knowledge outlined above that vagueness and continuity cannot be isolated from each other. On the one hand we have sign processes, within the competence of semiotics (as *System of Logic*) and on the other, continuity as a supreme law in the universe of phaneroscopic categories.

Semiotics itself, in its divisions and in the sign operations it defines, is the logic of vagueness, and it is in this sense that Peirce affirmed that he had elaborated such a logic. At the beginning, this logic was limited, as already shown, to the symbol, an unclear concept which was clarified processually, that is, to the extent to which Peirce arrived at the definition of the sign in its generality (the sign of natural language being only one among the possible signs of the global system of semiotics). Starting out from the particular term-object relationship, term-interpretant in particular, one arrives at the relationship between the sign and the object for which it stands, in particular the relationship between the sign and its interpretant (conceived as an integral part of the sign, united to the sign through the very act of interpretation). Vagueness thus comes about in the domain of the interpretant, a fact that has already led us to affirm that the opinion according to which “vagueness is a semantic notion” (with the addition that “It is deficiency of meaning,” as sustained by Kit Fine ([22], for instance) does not correspond to the essential determination of vagueness as a semiotic characteristic. It follows from Peirce’s analysis that vagueness is situated in the field of pragmatics, part of the generalized semiotic field that I have already defined [23]. The deficiency of meaning to which Fine and others refer to is *ambiguity* (so often confused with vagueness). The referential aspect (which the indefinite in breadth, hence the connotational aspect of the sign, represents) is not a source of vagueness. The mode in which signs are attached to objects, a mode represented by the referential aspect, is, in the final analysis, characteristic of the gnoseological moment. The structural aspect, stemming directly from the sign’s triadic-trichotomic structure (which the indefinite in depth, hence the connotational aspect of the sign, represents) shows both what vagueness is and what its logic is. It is a question of the way in which signs are connected to each other—a sign exists only in connection to another—of the way in which it participates in semioses, of the way they are interpreted, that is, of what characterizes the epistemological moment. This, in way of example, justifies the current discussions on the higher order of vagueness (M. Przelecky [24]), the meaning of which can, I believe, have more light shed upon it, considering the unity between vagueness and the continuum.

Recent theories, in the fields of science and the humanities, are characterized by, among other

things, a new epistemological condition, in particular by the integration of semiotic consciousness (hence including vagueness which in part of the latter) into scientific and philosophical practices. The evidencing of the semiotic aspect (of natural or formal languages, of the languages of communication or signification, etc.), that follows the prior event of demonstrating the structural aspect, corresponds to that discovery by the modern epistemology according to which “vagueness is not incompatible with precision” (Quine [25]). The need to reunite vagueness with the continuum is not therefore only the consequence of Peirce’s semiotic system—which is far from being universally accepted—but also a directly practical consequence necessitated by progressing from quantitative to qualitative evaluations.

### **Critical Remarks: Vagueness and Fuzziness**

Philosophers of language, logicians, and linguists accept that natural languages are vague. However, those who occupy themselves with general sign systems (in their quality as semioticians, logicians, or mathematicians) or with the study of specialized artificial systems (formal language, symbolic systems, institutionalized systems, etc.) start out from the need to define the source of vagueness and from the question of whether vagueness is an implicit characteristic of any semiotic representation or not. In this case too, opinions obviously do not coincide or even converge towards a commonly acceptable truth.

Two different positions should be considered as characteristic of the current evolution of vagueness. On one hand, the concept of semantic competence, which not only affirms the semantic condition of vagueness—an idea I consider out of harmony with Peirce’s fundamental concept of vagueness—but also ignores the referential relationship, eliminated the social aspect of meaning fulfillment in the broad sense. It also postulates (Chomsky, as well as Katz and Fodor) the clear distinction between what we know about the meaning of expressions and what we know about the empirical properties of things and phenomena. The second direction is represented by the new enunciations of the concept of competence (for instance, M. Creswell [26]). The capacity to decide between the truth or falsity of an expression is placed on the main level, hence the abstraction of all the aspects of meaning that are not connected to the expression is made. Putnam’s model [27] appeared as an alternative to the extent that it was characterized as an outright “realistic route.” Its argumentation is simple: we live in an epoch and in a society in which the principal of the division of labor functions, evident especially on the social and economic levels. The inference to linguistic activity is derived somewhat intuitively. On the basis of ordinary needs and interests, each person feels the necessity of learning the basic vocabulary. The necessity of setting up a method with whose help one can establish the relation between word and object (if the word is in univocal relationship with an object or in equivocal relationship with a number of objects) appears only to specialists. A relationship of cooperation (analogous to that in the labor process) is thus set up between experts and non-experts. Specialization, which corresponds to the growth of science and technology, brings with it the entrenchment of the division of linguistic activity. The mutual interest in cooperation between experts and non-experts is pragmatic and implies the social factor in language processes. Extension is socially defined through collective competence (which includes that of the experts). If we extend Putnam’s model to signs in general—and in his case as well as in Peirce’s, the beginning is made by considering natural languages—that is, if we extend the model to semiotic reality in its generality (which in the case of art, for example, would only be a confirmation, the

division of competence and the role of experts having been studied) we observe that the solution of describing semiotic processes (of language, especially) through formal language—Carnap’s line—is not sufficient since it cannot reflect both vagueness and the division of semiotic activity—a division that assumes more and more refined forms than Putnam supposes. A theory that considers both aspects is necessary, and such a theory can be merely contextual. (Putnam believes that the attempts made by D. Lewis [28] approach this ideal.) Here the problem that appears is one of pragmatic context, not semantic context, because, as I already have shown, vagueness, as well as the division of semiotic activity (I extend Putnam’s concept from the signs of language to the general system of signs), is determined on a pragmatic level. Context is not only semiotic, itself being sometimes vague (“its reference is often intrinsically vague itself,” J. Bar Hillel [29]). But any context, as I demonstrated in the definition of the semiotic field, can be represented by signs.

Let us not dwell on this. I have proposed to show that the logic of vagueness, as an implicit part of semiotics, is always found in association to the law of synechism, that is, to the doctrine of continuity. Thus Peirce becomes—I believe this opinion can be sustained—a predecessor of the view based on the model of fuzzy sets and especially their semiotic application in both analytical and synthesizing processes. Furthermore, the sign implies vagueness and the continuum, and this fact stems from Peirce’s general concept. Vagueness is modeled within the theory of fuzzy sets through the membership function that can gradually be brought to a higher degree of precision. This membership function has a certain similarity to the density of probability function when the set to which the latter refers is continuous, although the two are essentially different.

The ordinating concept of Peircean semiotics is the relation, the triadic-trichotomic structure preserving hierarchy from the model of the phaneroscopic categories. The preserving of hierarchy (that is, of a certain order relationship called isotony in mathematical language) makes formalization possible with the aid of the algebraic theory of categories. The objects of the fundamental mathematical category of the sign (see MacLane [30]) are represented by what is called *Firstness*, *Secondness*, and *Thirdness*. (I use the term “mathematical” in order to avoid confusion with Peirce’s metaphysical categories.) The nature of order relationship is expressed even through the names used. The relationship between these objects (classes, in fact) has, due to isotony (that is, due to the preservation of hierarchy indicated by name: Firstness before Secondness, Secondness before Thirdness, etc.), the nature of morphism. Here the structure of the relationship between objects, and not the objects themselves, are characteristic. As a result of this intrinsic condition of Peirce’s semiotic, a condition that can be strictly formally expressed through the category of sign classes, an important conclusion results: the infiniteness of interpretability, which stems from the sign’s vague condition, and indeterminateness, in connection to continuity, causes Peirce’s table of signs to make sense only to the extent it is understood as the structure of a continuously functioning system. Peirce himself did not observe this and consequently ordered the ten sign classes (2.264) according to an affinity criterion based on likeness. Actually, he did not observe that the ten sign classes were in reciprocal relationship, that some could be transformed into the others, nor how these transformations take place, although he defined the concept of semiosis (sign process), degenerative processes (leading to replicas) and generative processes (from a low degree of semioticity to a higher degree of semioticity). The application, in the abovementioned sense, of the mathematical theory of categories also permits the explanation of Peirce’s so-called

inconsistency which led him to write of ten sign classes on one occasion, then 28 (in his letter to Lady Welby of December 14, 1908 [31]) and even 66 classes. The explanation is that the ten classes correspond to the simple triadic structure of the phaneroscopic categories, or more precisely to what he called (also in a letter to Lady Welby, October 12, 1904) *cenopythagorean categories*.

Developing this first model, always under the control of the logic of relations—a logic that demonstrates the irreducibility of the triad into dyads or monads but assures the reducibility of higher forms into the triad—Peirce introduced, and explained in his letter to William James (March 14, 1909), the division of the object (immediate and dynamic), as well as of the interpretant (immediate, dynamic, final). Considering the morphism of this category (with no less than six objects), 28 classes actually result (ordinated independent sextuplets in conformity to the hierarchy given in the phaneroscopic categories). Another four trichotomies that Peirce suggested also explain the 66 classes mentioned, classes considered by the editors of his work as its final expression. So it is in no case a question of inconsistency, but an expression of the gradual perfecting of his typology of signs, to date not satisfactorily explained by anyone. Of course, it is captivating to follow the line of Peirce's reasoning, to reconstitute, by respecting the system's internal logic, some of the results that for a long time have been regarded with suspicion or presented as inconsistent (and discarded as such).

I have presented these results however, obviously important from a historical perspective, because they can be obtained only on the basis of the hypothesis I have enunciated: the unity between the logic of vagueness and the law of synechism, a hypothesis I have tried to demonstrate on the historical as well as methodical level. It is not only a problem of confirming Peirce and of consecrating him as one more precursor of the fuzzy set theory, but especially of developing his semiotic and putting it into operation. The definition of the dynamics of the sign table based on the consideration of morphisms from the cenopythagorean categories represents a first aspect. A second aspect is the observation that Peirce, basing himself on the unity of vague and continuous, intuited fuzzy relationships, that is he intuited multivalued relations and opened the way to the application of these relationships, belonging to his semiotics, to the dynamics of signs. The typology of the sign classes (the ten, the 28, the 66), as confirmed by the mathematical theory of categories (cf. Marty [32], Nadin [33]) should be understood as a network of fundamental reference points in the generalized semiotic field. Whenever this typology is transformed into an end in itself, it leads only to formalistic semiotics. To give a name to a sign (to identify it) does not solve the problem of the way it functions in the semiotic field. The sign can be conceived and interpreted only within the framework of the logic of vagueness and with the participation of the doctrine of the continuum.

Fuzzy categories, the extension of the mathematical concept of category, fulfill this desideratum and perfect Peirce's table of fundamental signs by realizing the image of the continuum, hence also the dynamics of sign processes. It is possible to go on, that is, to consider a suggestion of Peirce's (2.227) concerning the trichotomy of the icon, index and symbol, in which case again, the table of sign classes is *continuumized*, a result corresponding to the spirit of this semiotics based on the unity between vague and continuous [34]. In connection to this, it should be said that the concept of fuzzy sets, particularly of the *ensemble flou*, as they were introduced by Zadeh [35] and Gentilhomme [36] correspond to the reunion of borderline cases, as vagueness is

sometimes defined, with the doctrine of the continuum (which quantifies transition from one quality to another).

To quote Zadeh:

The fundamental concept in mathematics is that of a set—a collection of objects. We have been slow in coming to the realization that much, perhaps most, of human cognition and interaction with the outside world involves constructs which are not sets in the classical sense, but rather ‘fuzzy sets’ (or subsets), that is, classes with unsharp boundaries in which the transition from membership to non-membership is gradual rather than abrupt. Indeed, it may be argued that much of the logic of human reasoning is not the classical two-valued or even multivalued logic but a logic with fuzzy truths, fuzzy connectives, and fuzzy rules of inference [37].

The semiotic and dialogic nature of thought in Peirce’s conception and the model of multivalued logic demonstrated by Zadeh in his definition of fuzzy sets seem to be outright complementary components. It is clear that by joining all the theses I have presented above one can imagine a next level of fuzzy-fuzzy, etc. corresponding to the advancement from one level (or type) of indeterminacy to a higher one, obviously together with the image of the continuum extended to infinity. The exact treatment of the inexact, which many modern tendencies have programmatically assumed, thus becomes semiotically not only possible but also necessary.

I shall not dwell on the various attempts at fuzzy logic, which implicitly deal with some aspects of vagueness (Reiger [38], Zadeh [39], etc.). Not even the reservations made in regard to fuzzy logic (Morgan and Pelletier [40]) will be brought into discussion here, although in enunciating them, one touches upon problems concerning the condition of semiotics itself, hence Peirce’s basic doctrine (including the logic of vagueness). Our concern here has been to show the necessary relation between the components of this doctrine, a relation frequently ignored even though the price of this ignoring is the spoiling of the consistency of the semiotic procedure undertaken with the means of Peirce’s semiotic. In this study, I did not follow a formal path (although I presented results involving mathematical formalization [Nadin 41]), not because such a path is impossible or unpractical, nor because I fear instinctive rejection of mathematics or logics by semioticians. Rather, my intention was to suggest how congenial to our natural way of thinking and understanding are the logic of vagueness and the doctrine of continuity, hence how congenial the fuzzy approach is to us. Beyond the nonsystematic and often vague nature of Peirce’s formulations, this conclusion stands out with real clarity.

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## References and Notes

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- to this take the usual volume-paragraph form. As far as manuscripts are concerned, reference is made to Charles S. Peirce Papers in the Houghton Library by the numbers assigned in Robin's Annotated Catalogue of the Papers of Charles S. Peirce. References to Peirce's *New Elements of Mathematics*, edited by Carolyn Eisele, are by volume and page number.
2. Russell, Bertrand: "Vagueness,": in *Australian Journal of Psychology and Philosophy*, 1 (1923): 84-92.
  3. Wright, Crispin: "On the Coherence of Vague Predicates," in *Synthese*, 30 (1975): 325.
  4. Frege, Gottlob: "Über die wissenschaftliche Berechtigung einer Begriffsschrift," in *Zeitschrift für Philosophie und philosophische Kritik*, 81 (1882): 48-56.
  5. Carnap, Rudolf: *Meaning and Necessity. A study in Semantics and Modal Logic* (Chicago and London: The University of Chicago Press, 1947). In order to take vagueness into account while giving a formulation of the intension" of a predicate, "a pair of intensions F1, F2 must be stated: X has the disposition of ascribing affirmatively the predicate 'Q' to an object y if and only if y has F1; and the disposition of denying 'Q' for y if and only if y has F2. Thus, if y has neither F1 nor F2, X will give neither an affirmation nor a negative response; the property of having neither F1 nor F2, constitutes the zone of vagueness, which may be possibly empty" pp. 242-43.
  6. Wittgenstein, Ludwig: *Philosophical Investigations/Philosophische Untersuchungen* (Oxford: Blackwell, 1958).
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  12. Feibleman, James: *An Introduction to Peirce's Philosophy Interpreted as a System* (New York and London: Harper & Brother Publishers, 1946). Remarks on vagueness are made in a section devoted to "Critical Common-Sensism", pp. 310-16.
  13. Gallie, W.B.: *Peirce and Pragmatism* (Harmondsworth, Middlesex: Penguin Books, 1952 and *Peirce's Pragmaticism*," in *Studies in the Philosophy of Charles Sanders Peirce*, Philip P. Wiener and Frederic H. Young, eds. (Cambridge, Mass.: Harvard University Press, 1952), pp. 61-75. Approaching "The Problem of Vague Predicates" Gallie notices that Peirce, "in the Pragmaticism Papers, approaches the subject of vagueness from a number of different sides. He claims, for instance, that all our most deeply grounded and in practice indubitable beliefs are essentially vague (5.446); and this is true, in his opinion, both of particular "acritical" judgments of perception and of such highly general beliefs as that God (in some sense) is real, and that Nature is (in some sense) uniform. On the 'logic of vagueness' he has some remarkably acute and suggestive things to say, although nothing (as far as the Pragmaticism Papers or any other of his published writings show) that bears out his claim to have 'worked out the logic of vagueness in something like completeness'." p.66.

14. McKeon, K. Charles: "Peirce's Scotistic Realism," in *Studies in the Philosophy of Charles Sanders Peirce*, Philip P. Wiener and Frederic H. Young, eds. (Cambridge, Mass.: Harvard University Press, 1952), pp. 238-51.
15. Cohen, L., Jonathan: *The Diversity of Meaning* (London: Methuen & Co. 1962), chap. IX: "Meaning and Vagueness," pp. 265-77. "On C.S. Peirce's view it had been the law of non-contradiction rather than that of excluded middle which is restricted in scope by the phenomenon of universal vagueness," pp. 265-66. "Peirce seems to have concluded not that formal logic is intrinsically inapplicable to ordinary discourse but rather that a new logic, 'a logic of vagueness,' was required, which he said he had 'worked out with something like completeness'; and though Peirce's logic of vagueness has never been found various attempts have since been made to replace its loss," p. 266.
16. Hamilton, Sir William: *Lectures on Logic*. Eindhoven: H.L. Mansel and J. Veitch edit., 2nd vol.
17. Cf. Peirce, Ch., S.: "Upon Logical Comprehension and Extension," *Proceedings of the American Academy of Arts and Sciences*, vol. 7, Nov. 13, 1867, intended as Essay III of Search of Method and as chap. 15 of the Grand Logic.
18. "A symbol, in its reference to its object, has a triple reference: First, Its direct reference to its object, or the real things which it represents; Second, Its reference to its ground through its object, or the common characters of those objects; Third, Its reference to its interpretant through its object, or all the facts known about its object, cf. First, The informed breadth of the symbol; Second, The informed depth of the symbol." (2.408).
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