The Set of Signs

No matter what its type, a sign's principle function is to integrate. It represents and is defined by a triadic relation, the so-called triadic function of a sign, a function that is simultaneously the expression of the sign itself. Peirce's ideas and representations [1], which Morris [2] furthered, up to a point, and which the so-called Stuttgart School of Semiotic (Max Bense, Elisabeth Walther *et al,* preoccupied mainly with applications in various fields) [3] are relatively well known and highly productive. I shall attempt here an analysis from the perspective of set theory, developing new concepts such as analytic and synthetic semiotics, as well as generative (different from Bense's).

Take three non-empty sets: *Means* ("sign as a sign" in Peirce's terms, *Mittel* according to Bense), Objects *(Objekte)* and Interpretants *(Interpretanten)* [4]; or the set of *Repertory (Repertoire), Sphere of objects (Bereich)* and *Field of meanings (Bedeutung)* [5]. The sign is only--and no more than--the relation of a mean *(m),* an object *(o),* and an interpreter *(i);* that is : $S = (m, o, i) \in M \times O \times I$, which can be represented graphically

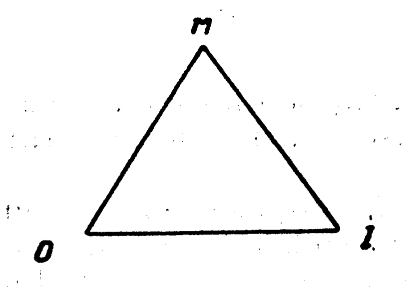


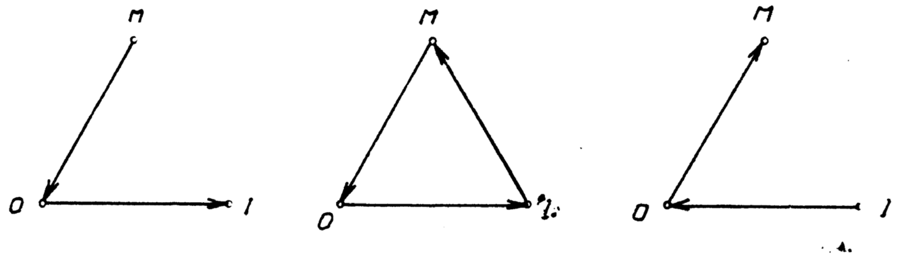
Figure 1.

and pictorially through the Venn diagram [6] of an intersection of the mentioned sets :

Figure 2.

It should be noted that the three sets are not univocally determined. From another perspective, the means can be objects, from another perspective, objects; objects are possible means; interpretants are possible means or objects. All this means is that the three sets are in fact two: *I* and *0,* that is, the object-subject relation, which imposes the need for a linking factor. The derivation of the set *M* corresponds to the introduction of the intentional factor (*praxis,* ***in*** a broad sense). Here the time factor is already implied. That is, the triadic scheme no longer appears as a given fact, but is continuously generated from the interaction of the three sets under consideration. In fact, Peirce's original model is made evident through the sign as representamen, its integrating nature: "the representative character of a representamen by representing a parallelism in something else."

The dynamic condition, which the triadic function presupposes in Peirce's concept, i.e., through the direction in the generation from Firstness to Secondness to Thirdness, was pictured in graphs [8]:



a) generative graph; $M \to O \to I$ b) thetic graph; $I \to M \to O$ c) degenerated graph; $I \to O \to M$

Fig. 3

In the same manner, the semiotic matrix applications (cf. E. Walther) established through arithmetic notation (Bense) yield a similar result.

The representation I propose, which is in agreement with the manner in which Peirce himself approached the question of the definition of the sign, retains the three sets (as shown in fig. 2, alongside the Venn diagram) and implies the two operations that this envisages (denomination and signification). But semiotics appears on the proposed scheme together with other fields of interference (intersections) such as:

$M \cup O \setminus I$; $I \cup O \setminus M$; $I \cup M \setminus O$ or even $M \setminus O \cap I$; $O \setminus I \cap M$; $I \setminus M \cap O$.

The first three are in fact expressions of the syntactic, the semantic, and the pragmatic *per se.* They belong to semiotics and constitute its levels, but they cannot really represent it except in the case in which the three sets are identical (likewise when the three circles representing them are superimposed or joined). This leads us to the notion that the sign is primarily the *relation* between its three necessary elements; moreover, as such a relation, its action is integrating. Peirce's categorical system imposes such a quality.

No other relation between two of the three elements can represent the sign or permit its realization. If, for example, as it so often happens, someone confuses the sign with its mean, this leads back to a consideration of the mean as the set (repertory):

a) $M_m = \{(m,o,i): o \in O, i \in I\}$

And the other two possibilities are open:

b) $O_o = \{(m,o,i): m \in M, i \in I\}$; $I_i = \{(m,o,i): m \in M, o \in O\}$.

In the first case (a), we actually have the set of all signs with a common mean. In the second, we have the set of signs representing the same object. In the third is the set of signs related to one and only one interpreter. In Figure 2, these extreme cases are evident. In each case, semiotics is reduced from the *surface* of all possible sign relations to the intersection of this surface ad one of its definition sets. The product of these three sets mentioned above is set S--the set of signs. The power of each of these sets, which are equivalent, is in the category of the power of natural numbers (aleph-zero, or X_0). This result will permit the determination of the power of the set of signs (according to a method different from that of Hermes and Scholz [10]).

In Fig. 2, one can also decipher the way in which a sign is *constituted* and *instituted.* The semioses (sign processes) are here somehow approximated, but the operations (adjunction, superization, and iteration) seem relatively clear, especially when we keep in mind the operations with elements of a set.

Each sign appears in the space $M \times 0 \times I$ and therefore continuously generates a relation of signs. In this way it results that the introduction (constitution) of a sign in fact means the opening towards the sequence of signs, because each sign is open on each of its sides. It is true that the basic representation (fig. 1) is more adequate in showing how and when adjunction, iteration, and superization (in Bense's sense) occur. But sign processes appear distinctly clear when the sign's operation is considered as an intersection of sets. Likewise, the adjunction corresponds to a unique object and to a series of pairs $m, i,$ therefore to a generated set :

$$O_o = \{(m,o,i): m \in M, i \in I\}$$

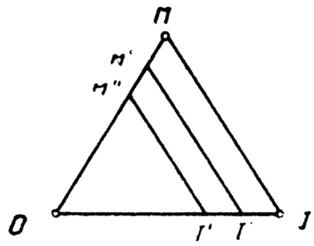


Figure 4.

Similarly, superization corresponds to a unique mean and to a series of pairs $o, i,$ therefore to the generated set:

$$M_m = \{(m,o,i): o \in O, i \in I$$

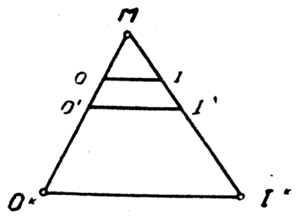


Fig. 5

Figure 5.

and iteration: $I_i = \{(m,o,i): m \in M, o \in O.$

The formalization of a sign's operation in the terms of the set theory (after it has been reproduced in graphic representations [11] or in matrix calculus [12]) has the advantage of proposing a link between semiotics as a method of analysis--or what I shall call *analytical semiotics*--and synthetic semiotics, and will lead to a suitable model of generative semiotics.

If the sign is an element of the set $S = (m, o, i) \in M \times 0 \times I$ and if we consider S a given nonempty set (signs in a given domain, therefore a type of applied semiotics), and $C \subset S_N$ is a field of criteria (for example, the criteria of relating the sign to $M, 0, I,$ therefore $n = 3$ in Peirce's triadic spirit), then an analytical semiotics is endowed with the function $S_A : S \rightarrow C$, that is, S_A is defined on the set S with values in the field of criteria $C.$ It can be seen that through the criteria of relating the sign to the constituent elements of the sign function, Peirce imposed a type of semiotics (S_A). In fact, he did not exhaust all the types of semiotics. (The analysis can also be of the form of a sign, of significance, of communication, or of the form-function relation, among others.) If, for example, someone were to propose other criteria—Klaus [13] divided the object relation and proposed a so-called sigmatic aspect of the sign (in this case, evidently, (n = 4)--another type of semiotics would result. Peirce's attempt at a consistent sign theory is evident through the way he defined the types of signs (as related to Mean, Object, Interpretant, that is, the triplets Quali-, Sin-, and Legisign; Icon, Index, Symbol; and Rhema, Dicent, and Argument), but still the class of all possible combinations (amounting to 27) exceeds the class of actual possible combinations (amounting to 10). These are: six Rhematic, three Dicentic and one Argument (due to requirements other than those related to ordering).

The degeneration of the sign, in the transition from signs (or class of signs) of higher semioticity to signs (or class of signs) of lower semioticity (that is, from consciousness to objective reality as expressed through signs) is reflected in the analytical table and expressed in the formula of semiotic inclusion (the syntactic represents, by its own definition, the level of highest semioticity). The fact that the sign is a *rule* (of the relation) leads us to the idea that semiotic is an *axiomatic* system. This cannot prevent us from recalling that the *system of criteria Peirce (C_P)* is not powerful enough (it is a relatively "weak grammar," cf. Chomsky). Quali- implies the Sin- and Legisigns; Dicent implies the Argument, which leads to, among other things, the 10 classes possible in this type of analytical semiotics. Stricter criteria lead to the more limited generative series ("strong grammars").

On the other hand, each time semiotics is used in an analytical attempt, it becomes evident that the basic triadic relation need not be considered as the structural rule of all semiotics, but only as one of its possible axioms. Even the three levels of semiotic deriving from the given syntactic-semantic-pragmatic axiom were put in doubt. Hermes and Schröter accept only two levels of semiotics viewed from a mathematical perspective. Other theoreticians later introduced a fourth level, taking into consideration the general theory of action, i.e., praxiology, involved in semiotics. The validity of the syntactics-semantics-pragmatics trichotomy was also discussed from a linguistic perspective (at the *International Working Symposium on the Pragmatics of Natural Languages,* 1970 [14]). It must be recalled that Morris' analysis, which forms the basis of the trichotomic model, has affected Peirce's model of semiosis. He abstracted three kinds of dichotomic relations for study: between sign and interpreter (pragmatics); between sign and 'designation' (object denoted, semantics); and between and sign and sign (syntactics). He was correct in relating the trichotomic distinction to fundamental aspects of communication. However, it has been shown (Hans-Heinrich Lieb at the symposium above mentioned) that for a fruitful formulation of the distinction, it is necessary to develop much more elaborate theories of communication. I shall not do this here, but I wanted to introduce this aspect into the discussion precisely in order to propose the idea of an axiomatic constitution of semiotics.

An analytical semiotics determines the place of any type of sign in the space of the proposed criteria (in Peirce's case, in the three-dimensional field of the reference values $(M, 0, I)$. In this case S_A : $S \rightarrow C$, when $C \subset S_n$ (i.e., C is implied in a finite space of criteria). Obviously, sometimes two different signs deal with the same object, without necessarily being identical because of this. They can have a common coordinate (o_i), or even two, within the field of reference. Again, the scheme of the intersection of nonempty sets $M \times 0 \times I$ is suggestive of the above.

The question arises whether function S_A (or application S_A, which I call analytical semiotics) is reversible. In the affirmative case, we have $S_S : S \rightarrow C$, which I shall call synthetic semiotics. A function is reversible if and only if it is bijective (that is, injective and surjective).* In this case the demonstration is simple : for the function $S_A : S \rightarrow C$, we have $S_A(s') = S_A(s'')$ only when $s' = s''$ (I) because as was here determined, $s = (m, o, i) \in M \times 0 \times I$; then, $c = S_A(s)$ (II), because each criterion is a coordinate in the space S_A. Being bijective, the sets S and C are *equipollent.* Since conditions II and I are fulfilled, it follows that the application (function) $S_E : S \rightarrow C$ also exists: $S_E = S_A^{-1}$, corresponding to the uniting of a coordinate in the space of criteria of one or more signs ("classes" in Peirce's terminology). This takes us back to the possibility of the synthesis of a sign with prescribed properties, or more precisely, the synthesis of a group of signs with a given property or a set of properties (the typical case in problems of design and visual communication).

Analytical semiotics is univocal. In relation to an adopted system of criteria, a sign (or an ensemble of signs) presents itself as having a determined quality. Synthetic semiotics is equivocal. Its definition presupposes rules of formation from the triadic sign relation to the three fundamental operations (adjunction, iteration, superization), as well as to their possible combinations. In fact, an equivalent exists between the synthetic function and the three graphs (fig. 3) of the generation of a sign, the function obviously being more encompassing. Finally, Bense's concept [16] concerning the distinction between internal (from Qualisign through Sinsign to Legisign) and external superization is implied in the synthetic function.

If we could imagine a sign "device" [17] (not necessarily the type represented by a computer), all that would remain would be the consideration of a generative semiotics (perhaps considered even as a cybernetic system). The problem is reduced to the determination of the succession of the operations through which a set $C \subset C'$ so that $S_G : C' \to S_A$, in which $S_G(c) \in S_s(c)$, that is, an indeterminate algorithm, could be generated. In this case, S_G is a generative semiotics.

The need for a subset $C' \subset C$ is based strictly on practical reasons. If the device is ideally workable, then generative semiotic is identical with synthetic semiotics. In order to generate the signs of Peirce's semiotic, any generative operation must reproduce the basic triad $M \times 0 \times I$, of course capable of having any direction imposed from the beginning ($M \to 0 \to I$) that is, Firstness-Secondness-Thirdness), or determined randomly (or quasi-randomly).

The set of signs is not homogenous. The power of set S (as shown also by Hermes and Scholz) is in the category of the power of natural numbers (aleph-zero, or X_0). The power of the set of criteria is finite and determined through the particular axiom of every semiotic. (Peirce's semiotic is defined through $C^{=}$= 3). The higher the power of the set of criteria is, the more determined the signs become. At the extreme $C^{=} = S^{=}$ (i.e., the power of the set of criteria equals the power of the set of signs), every sign ceases to exist. At the other extreme, the signs become less and less determined. In a way, this is the case in such a sign reality as art. Here again we see in which way the signs expresses the level of human integration.

In general, any synthetic semiotics is a semiotics of finite power. Semiotic analysis also requires a reduction from the infinite (or the power of the continuum) to the infinite. The set of signs could be analyzed from the perspective of ordering (order relations are anti-reflexive, transitive, and asymmetrical); and I could propose operations on order relations. In this case, the possibility of proving the classification by means of combining various order relations is opened. Here I only suggest that a set theory attempt in semiotics opens several possibilities for checking recent conclusions regarding semiotics.

The separation from context--such a controversial point--is basically a diagonal process. This interpretation must be retained, as must the significance of the way in which the set of signs is established. Introducing a third set (the means) into the space of intersection of the two sets of reality (objects and subjects), semiotics has allowed for the expression of the integrated condition of human beings (as subject, but also object unto himself) in the natural and socio-historical reality to which they belong. Analytic and semiotic synthesis (generation) express nothing other than the degree of' integration.

* The mapping α: $M \to L$ is called surjective if each element y from L has a pre-image. In this case, it is also said that M is mapped onto L . The mapping α: $M \to L$ is called injective if each element $y \in L$ has at most one pre-image. If the mapping α: $M \to L$ is simultaneously surjective and injective, it is called bijective [15].

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